DETERMINATION OF HEAT-TRANSFER COEFFICIENTS IN POROUS MEDIA

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We consider cooling by transpiration. We discuss methods of determining the volume heat-transfer coefficient using a two-temperature model and the surface heat-transfer coefficient on entry into a porous wall.

The development of new technology using increasing heat loads has stimulated the search for effective methods of increasing the rate of heat transfer. One of the promising directions in the solution of this problem is the use of porous materials in the elements of power devices. Cooling by transpiration has been used successfully to provide shielding from convective or radiative heat fluxes. The heat-shielding mechanism is as follows. The injection of cooling air through a porous medium creates a screen along a body in the flow and heat is drawn away from the skeleton of the porous material. The volume heat-transfer coefficient a_V (which is assumed to be given) is normally used in the mathematical modeling of the thermal interaction between a gas and the skeleton of the porous medium. However, analysis of a number of experimental and theoretical papers on the basis of this characteristic of heat transfer in porous media [1-5] shows serious disagreements between the different results. As noted in [1, 2], the transfer of heat from the surfaces of the pores to the injected medium is characterized by the surface heat-transfer coefficient α_S, and the following relation exists between the two types of heat-transfer coefficients

$$\alpha_V = \alpha_S \frac{S}{V}, \tag{1}$$

where S is the active surface area of the pores. However, the dependence (1) has not been widely used because of the difficulty of determining the active surface area of the pores in realistic porous media. The difficulty is due to the discreate nature of the medium, different shapes of the elementary particles, and the fact that it is not possible to measure exactly the temperature of the skeleton and the gas inside the pores. The formulation of boundary conditions on entry into the porous medium is also not trivial, since there does not exist a theoretical relation which can be used to calculate the surface heat-transfer coefficient α_w [2].

We consider a uniform porous material in the form of a cylindrical region with spherical and cylindrical particles, fibers, and strands of fibers serving as the structural elements. It is assumed that the elementary particles have point contacts with one another and that the cooling gas is injected and the heat loads are applied on the ends of the region. Then, taking into account the fact that the fraction of blind and closed pores is small, one can calculate analytically the area of the active pore surface for porous media with different elementary particles. The expressions for the different particles are given in Table 1.

Assuming the volume fractions of the skeleton V_S are equal, we obtain a simple relation expressing the equivalence of the active surfaces:

$$\frac{3}{R_{\text{sph}}} = \frac{2(L+R)}{LR} = \frac{2}{R_f} = \frac{2}{\left(\frac{N-m}{2}+1\right)r},$$

where in the case of a strand of N fibers it was noted that an internal fiber (there are m internal fibers in the strand) does not participate in convective heat transfer, and therefore should not contribute to (1). Therefore the characteristic dimension of a strand of

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TABLE 1. Transition from Porous Material to Perforated Region

Structural element of the porous material	Active surface areas of the pores, m^2	Dependence of the channel radius R_{ch}, m, on the particle size	Volume Nusselt number and the number of channels in the equivalent region
Spherical particle	$S=\frac{3Vs}{R_{sph}}$	$R_{ch}=\frac{2}{3}\varphi R_{sph}$	
Cylindrical particle	$S=\frac{2(L+R)}{LR}Vs$	$R_{ch}=\varphi\frac{LR}{L+R}$	$Nu_V=2\Pi Nu_S$
Fiber	$S=\frac{2Vs}{R_f}$	$R_{ch}=\varphi R_f$	$n=\frac{1}{\Pi}\left(\frac{S}{F}\right)^2$
Strand of fibers	$S=\frac{2Vs}{\left(\frac{N-m}{2}+1\right)r}$	$R_{ch}=\varphi\left(\frac{N-m}{2}+1\right)r$	

fibers was taken to be the radius of an effective monofilament whose surface area is equal to the active area of the outer fibers in the bundle.

Real porous media are normally studied using certain physical models such as a system of small spherical particles with different types of packing or a set of transmitting cylindrical capillaries. We consider the simplest model of a porous medium as a bundle of right circular cylindrical channels. In contrast to [1], where a model of this type was used to make the transition from porous medium to a perforated region with cylindrical pores, we introduce the additional condition that the active surface area of the pores of the model must be equal to that of the real medium. In this way the individuality or specific nature of the original material is taken into account in a more rigorous way. In Table 1 this transition is expressed in the form of a dependence of the radius of a cylindrical channel on the dimensions of the elementary particles, as well as a formula for the number of capillaries n, which is needed to establish the density of perforations. In addition, to describe thermal processes in the equivalent model medium on the basis of (1), we constructed a dimensionless equation relating the volume Nusselt number with the porosity of the material and the surface number Nu_S:

$$Nu_V = 2\Pi Nu_S, \tag{2}$$

where the channel radius is taken as the characteristic linear dimension. We note that in a number of cases this formula can be used to determine the rate of volume heat transfer in a porous medium without the specification of Nu_S, since data are available in the literature on heat transfer in pipes and channels both in the inlet and downstream regions. In addition, the dependence (2) not only does not contradict (1), it supplements it by introducing an important parameter: the porosity Π. The relation (2) supports the conclusion of the experimental study [6] that the porosity has an important effect on α_V.

In [4] the following values of the volume heat-transfer coefficient were obtained from the solution of the inverse problem of heat conduction for three different methods of measuring the temperature in the porous medium: $\alpha_V = 2.668\cdot10^6$; $7.251\cdot10^5$; $1.452\cdot10^6$ W/(m³·°K). Making the transition from this porous medium with spherical particles as structural elements with the parameters $D = 5\cdot10^{-2}$ m, $H = 4\cdot10^{-3}$ m, $d = 6.3\cdot10^{-4}$ m, $\Pi = 0.3455$, $M = 25.65\cdot10^{-3}$ kg/sec [4] to a model perforated region with cylindrical capillaries using the appropriate equations from Table 1, we obtain: $R_{ch} = 1.109\cdot10^{-4}$ m, $S = 4.893\cdot10^{-2}$ m², $V = 7.854\cdot10^{-6}$ m³, $F = 6.283\cdot10^{-4}$ m², $n = 17554$. The following results were obtained for α_V for the model perforated region using the dimensionless equation (2) and the known values of the Nusselt number in the stabilized sections of heat transfer and the hydrodynamic equations for flow inside a pipe at constant temperature T_w or heat flux q_w on the wall [7]. T_w = const: T_w = const $Nu_S = 3.586$; $\alpha_V = 2.75\cdot10^6$; q_w = const: $Nu_S = 4.365$; $\alpha_V = 3.28\cdot10^6$.

By comparing the values of α_V obtained from the solution of the inverse problem to those calculated from (2), it follows that the best agreement occurs for constant temperature on the wall, which corresponds to the conditions of the experiment in [4]. We note that this example reflects the existence of a lower limit of applicability of (2) because of the use of the asymptotic value of Nu_S in the downstream region. In the general case of unsteady convective heat transfer in a porous medium, the internal heat transfer is determined by the value of Nu_S on the initial hydrodynamic sections of the model capillary.

The mathematical modeling of cooling by transpiration assumes that the specific boundary conditions are also taken into account in the formulation of the problem. However, the complicated nature of heat transfer on entry into the porous wall is reflected in the diversity of conditions used. The conditions suggested by Polyaev, Maiorov, and Vasil'ev [2] are the most justified. The groups of conditions are distinguished by the direction of injection of the gas. For injection of gas along the normal to the surface of the the porous medium

$$\lambda \frac{dT}{dx} = Gc(t - t_0); \tag{3}$$

$$\lambda \frac{dT}{dx} = \alpha_w (T - t_0) \quad \text{or} \quad \frac{\alpha_w}{Gc} = \frac{t - t_0}{T - t_0} = \mathrm{St}_w, \tag{4}$$

and for flow along the porous wall

$$\lambda \frac{dT}{dx} = \alpha (T - t_0); \tag{5}$$

$$\lambda \frac{dT}{dx} = Gc(t - t_0) \quad \text{or} \quad \frac{\alpha}{Gc} = \mathrm{St}_w, \tag{6}$$

where α is the effective heat-transfer coefficient from the permeable surface to the cooling gas flowing into it; St_w is the Stanton heat-transfer number on entry into the porous wall. It characterizes the relative heating of the gas. The parameters α_w and α are not determined theoretically, but rather are fit to the data of a small number of experimental studies on the inlet into a porous wall in the form of the dimensionless relation $\mathrm{Nu}_w = \mathrm{Pe}^n$, $n \leq 1$.

By analyzing the structure of the porous medium, we can identify in each specific case the characteristic element of the medium with the heat transfer and hydrodynamic processes intrinsic to it. Repetition of the element then determins the structure of the porous medium and also the entire heat transfer and flow on entry into the porous wall. Spherical particles and cylindrical fibers were chosen as the most typical of such elements. Also we used the results of theoretical studies based on the solutions of the equations in the boundary-layer approximation and written in the form of dimensionless heat-transfer relations for flow around the spheres and cylindrical tubes. The maximum values of the Nusselt number were observed at the stopping points [7, 8]:

$$\mathrm{Nu} = 1{,}315\,\mathrm{Re}^{0,5}\,\mathrm{Pr}^{0,4} \quad \text{(sphere)} \tag{7}$$

$$\mathrm{Nu} = 1{,}14\,\mathrm{Re}^{0,5}\,\mathrm{Pr}^{0,4} \quad \text{(cylinder)}, \tag{8}$$

where the diameter of the sphere and cylinder were taken as the characteristic linear dimensions. The transformation (7) and (8) used in (4) and (6) gives

$$\mathrm{St}_w = \frac{A}{\mathrm{Re}^{0,5}\,\mathrm{Pr}^{0,6}}, \tag{9}$$

where A = 1.315 in the case of spherical particles and A = 1.14 for cylindrical particles. The validity of this transformation from a single element to an ensemble of particles on the surface of the porous medium is based primarily on the satisfactory qualitative agreement between (9) and the experimental data on entry into the porous wall [2] (see Fig. 1). Curves 5 and 6 were constructed using (9) and show the variation of St_w for porous media consisting of spherical particles and fibers, respectively. In [4] the surface heat-transfer coefficient on entry into the porous wall was determined as α_w = 350 W/(m^2·K), in addition to the dermination of the volume coefficient. The calculated value using (9) and the data for air c = 1005.7 J/(kg·K); $\mu = 1.983 \cdot 10^{-5}$ kg/(m·sec); Pr = 0.708; T_0 = 300 K was similar: α_w = 353.3 W/(m^2·K). An analogous comparison with [9], in which the heat-transfer coefficient on entry was α_w = 1160, gave satisfactory agreement with α_w = 1043 according to (9). The condition $\mathrm{St}_w \leq 1$ on the relative heating of the gas on entry into the porous medium, which follows from (4) and (6), was also true for (9).

The consistency of the theoretical results with the experimental data suggests methods of determining the volume and surface heat-transfer coefficients in the design of porous materials and of calculating their heat-transfer properties. The results can be used to improve the technology of producing porous materials.

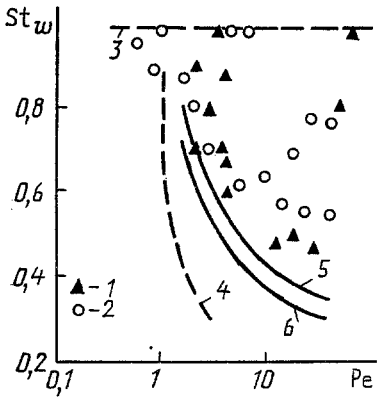

Fig. 1. Comparison of the theoretical and experimental results for St_w: data of [2] for d = 100 μm (1); d = 350 μm (2); St_w = 1 (3); $St = Pe^{-0.765}$ (4); Eq. (9) with A = 1.315 (5); Eq. (9) with A = 1.14 (6).

NOMENCLATURE

R_{sph}, R and L, R_f, r, linear dimensions of particles in the form of spheres, cylinders, single fibers, and bundles of fibers; Nu_V, volume Nusselt number; λ, thermal conductivity of the cooling gas; Nu_w, Nusselt number on entry into the porous wall; Pe, Re, Pr, Peclet number, Reynolds number, Prandtl number; μ, dynamical coefficient of viscosity; T, T_0, temperature of the skeleton and initial gas temperature; d, particle diameter; D, H, F, V, diameter, thickness, lateral surface area, and volume of the sample; M, mass flow rate of the gas; $\phi = \Pi/(1 - \Pi)$; G, specific mass flow rate; c, specific heat capacity of the gas; t, t_0, temperature of the gas on entry and before entry into the porous material.

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